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enables us to illustrate clearly what is meant in astronomy by the parallelism of the earth's axis, since the model may be carried by the hand slowly round in any circular or elliptic orbit, without any perceptible deviation of the axis from its original direction. But when the centre of gravity is brought slightly below the point of support, we are then enabled to show the deviation from parallelism which arises in the direction of the earth's axis after a long period of years, the same motion exhibiting the precession of the equinoxes. With the centre of gravity so placed, if the ball is made to rotate in the direction marked by the upper arrow on the figure, or from west to east, the equinoctial point E is observed to move slowly in the direction marked by the lower arrow from east to west. The latter motion may be made as slow as we please, so as to approach within any degree of closeness the exceedingly slow precessional movement of the earth's equator."

In another part of his paper the author makes the following remark:—

"If we next load the sphere on one side *very slightly* by any means, we obtain an illustration of the nutation of the earth's axis, the axis making a multitude of minute conical revolutions round the circumference of the greater conical revolution."

Note on the Method of Computing the Moon's Parallax.
By Mr. Sang.

In deducing the true from the apparent place of a heavenly body, it is usual to correct the observed altitude for the effect of atmospheric refraction, and to use the altitude thus corrected as the argument for the parallax; the sine of the parallax being supposed to be proportional to the sine of the corrected zenith distance.

This operation is founded on the hypothesis, that on correcting the observed altitude for refraction we obtain that altitude at which the planet would have been seen from the observatory if there had been no atmosphere. This hypothesis, however, is incorrect, as is evident when we consider that the ray of light, impinging on the upper surface of the atmosphere, is gradually bent downwards, and reaches the eye after having traced a curved path through the air. The angle of refraction is that angle which the first direction of the light makes with its direction when just entering the telescope; and, therefore, on correcting the observed altitude for refraction we obtain the direction, not of the original unrefracted ray, but of a line drawn parallel to it through the place of observation, which line cannot pass truly through the planet.

Thus if A be the observatory on the earth's surface, M the position of a celestial body, and M B the ray of light entering the atmosphere at B and then describing the curved line B A, the

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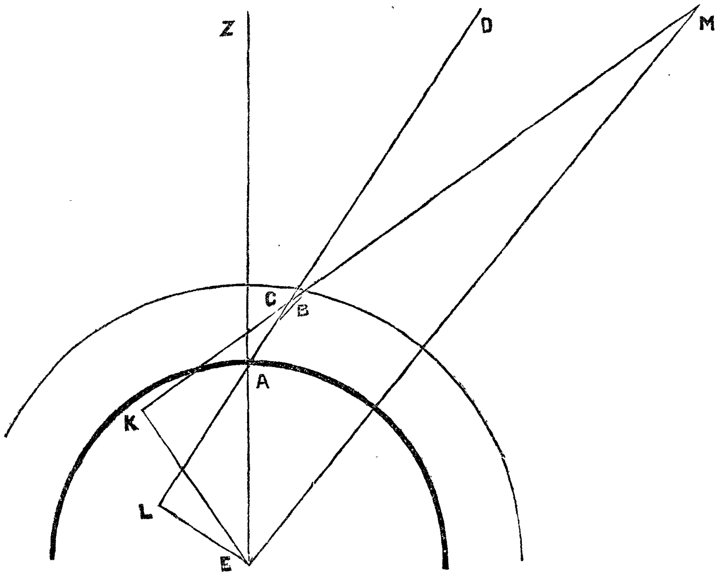
line A D drawn to touch that curve at A gives the apparent position of the planet; and if we continue M B to meet the tangent in C, M C D is the angle of refraction. When, therefore, we add the refraction D C M to the observed zenith distance Z A D we obtain the zenith distance of a line drawn through A parallel to C M; and this corrected zenith distance exceeds that which the planet would have had if there had been no atmosphere by the angle A M B, under which the curved path B A would be seen from the planet.

In the case of the primary planets this angle A M B is much too small to be taken into consideration, but in the case of the earth's satellite it may possibly amount to something appreciable. My object in the present paper is to estimate the amount of error caused by neglecting this angle.

At first it would seem that for this purpose we must know the nature of the curve B A; and that thus all the uncertainties which perplex us in regard to the gradual attenuation of the air would complicate our inquiry. But, fortunately, it turns out that these considerations are eliminated, and that the strict computation of the parallax is more simple and more direct than the ordinary approximate computations.

Having joined M with E, the centre of the earth, E M B is the true parallax. Continue M B and D A to meet perpendiculars let fall upon them from E. Then it is a well-known property of rays refracted by concentric strata that the ratio of these perpendiculars E K and E L is that of the indices of refraction of the strata at B and A inversely.



Now the index of refraction of the air at A can be obtained by direct experiment, or can be computed from the states of the barometer and thermometer, so that the ratio of E K to E L is known. Let this ratio be denoted by i.

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Then we have

$$\sin \mathrm{EMK} = \frac{\mathrm{EK}}{\mathrm{EM}} = i\frac{\mathrm{EL}}{\mathrm{EM}} = i\frac{\mathrm{EA}}{\mathrm{EM}} \sin \mathrm{ZAD}.$$

But $\frac{\mathrm{EA}}{\mathrm{EM}}$ is the sine of the moon's horizontal parallax, on the supposition of there being no atmosphere; and, therefore, it appears that the sine of the actual parallax is obtained by multiplying the sine of the horizontal parallax by the sine of the observed zenith distance and by the index of refraction of the air.

The true argument for parallax is thus the observed, and not the corrected, altitude, the formula being

$$\sin \text{par.} = i \,.\, \sin \text{Hor. Par.} \,.\, \sin \text{app. Zen. Dist.}$$

On comparing this formula with the usual one, we find that the coefficient *sin corrected Zen. Dist.* has been replaced by *i . sin observed Zen. Dist.;* these two coefficients would be identical if the strata of the atmosphere were plane instead of spherical, so that the true computation must give a result differing more and more from the usual one as the angle AEB becomes larger, and we may expect the greatest deviation to be when the moon is in the horizon. When the apparent zenith distance is 90° the corrected is about 90° 33′, and the sine of the corrected zenith distance, instead of being greater, is actually less than that of the apparent. For the purpose of roughly estimating the inaccuracy of the common formula we may take these sines as equal to each other; now the index of refraction of air at its mean density is $\frac{3405}{3404}$ nearly; and, therefore, for planets apparently on the horizon, the parallax exceeds that quantity which is given in the almanacs as Hor. Par. by $\frac{1}{3404}$th part of itself; which gives in the case of the moon at its mean distance almost exactly one second of arc.

The strict computation of the parallax would be at once effected, if, instead of the horizontal parallax, as given in the almanac, which is half of the angle under which the earth would be seen from the moon were there no atmosphere, we had the true horizontal parallax, or that angle which the earth, as magnified through the atmosphere, actually does subtend at the moon. The sine of this augmented horizontal parallax would then have to be multiplied by the sine of the observed zenith distance. It is true that such a table would serve strictly only for observations made in the mean state of the air; but the variations in the index of refraction scarcely affect the result; a change of one inch in the barometer only producing a change of 0″·03 in the horizontal parallax. With the present arrangement we can easily obtain the strict result by adding 1285 to the seven place logarithm of the parallax.

From this investigation it appears that, in reducing observations made on the occultation of fixed stars by the moon, it is scarcely necessary to take the actual amount of refraction into account at all: at most, in observations on moon-culminating stars, the differential refraction is required.